Differential Equations Proof(?)

Suppose we want to solve the differential equation

$$\int_0^x f(y)\,dy = f(x) - 1$$

We can rearrange and simplify this to $1 = f\big(1 - \int\big)$, and dividing yields $f = \frac{1}{1-\int}$. From the series expansion of the right hand side, we can see that

$$f = 1 + \int + \int^2 + \int^3 + \ldots$$

Recall $\int = \int_0^x dx_0 = x$ and exponentiation gives us

$$\Big(\int\Big)^n = \int_0^x \int_0^{x_{n-1}} \ldots \int_0^{x_1} 1\,dx_0 \ldots dx_{n-1} = \frac{x^n}{n!}$$

Therefore, we have our solution

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots = e^x$$

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